FULLY WORKED SOLUTIONS

Context 10: Medical physics

Chapter 23: Technological methods of medical diagnosis

Chapter questions

1. $v = f\lambda$

$330 = 256\lambda$

$\lambda = 330 \div 256$

$= 1.29$ m

2. $v = f\lambda$

$330 = 25\,000\lambda$

$\lambda = 330 \div 25\,000$

$= 0.013$ m

$= 1.3$ cm

3. $v = f\lambda$

$v = 30\,000 \times 0.003$

$= 90\ \text{ms}^{-1}$

4. (a) $v = f\lambda$

$1570 = 2.5 \times 10^6\ \lambda$

$\lambda = 6.28 \times 10^{-4}$

(b) $Z = \rho v$

$= 1050 \times 1570$

$= 1.65 \times 10^6\ \text{kg m}^{-2}\ \text{s}^{-1}$

5. $Z = \rho v$

$= 1140 \times 1620$

$= 1.85 \times 10^6\ \text{kg m}^{-2}\ \text{s}^{-1}$

6. $I_r \div I_o = (Z_1 - Z_2)^2 \div (Z_1 + Z_2)^2$

$Z_{air} = 1.3 \times 330$

$= 429$

$Z_{fat} = 1.38 \times 10^6$

$I_r \div I_o = 0.999$

99.9% of the ultrasound is reflected.

7. $I_r \div I_o = (Z_1 - Z_2)^2 \div (Z_1 + Z_2)^2$

$Z_{bone} = 1400 \times 4080 = 5.712 \times 10^6$

$Z_{muscle} = 1075 \times 1590 = 1.709 \times 10^6$

$I_r \div I_o = 0.29$

29% of the ultrasound is reflected. Therefore 71% is transmitted through the bone.

8. Soft X-rays ,with lower frequency and less energy than hard X-rays, will not penetrate very far into the body. Some of them need to go through the body so that an image can form.

9. Soft X-rays do not pass through the body and so are not involved in producing the image. They are absorbed by the upper layers of skin, where they may cause damage. Hence they are removed from the X-ray beam.

10. When we see an X-ray of a bone, the X-rays have not passed through the bone but have passed through the surrounding tissue and exposed the photographic plate around the bone.

12. X-rays and ultrasound are less expensive than CT scans. They are quick to administer and so are often used to obtain an initial diagnosis of a problem before further testing using CT is undertaken if necessary.

13. X-rays will not readily penetrate bone, so a conventional X-ray of the head would provide an image of the skull that surrounds the brain rather than the brain.

Review questions

1. Ultrasound is sound above 20 000 Hz frequency, whereas sound for normal hearing in humans is of frequency below 20 000 Hz. 20 000 Hz is at the limit of human hearing.
2. High frequency ultrasound does not penetrate very deeply into the body, so it could not be used to image organs very far below the skin.
3. An ultrasound A-scan is a range-measuring system in which an ultrasonic pulse is sent into the body in one line and the time for the pulse to be reflected from an interface in the body is measured. The intensities of the reflected signals are plotted as a function of time, and from this information the position of features inside the body can be found. For a B-scan, a pulse is sent into the body along one line and the reflected pulses recorded as dots, the brightest corresponding to the signal of greatest intensity. By sending pulses into the body at different angles a series of B-scans can be used to build up a 2-D picture of a cross-section through the body.
4. (a) Electrons are emitted from a heated filament in a highly evacuated tube. The heated filament is the cathode. A very high potential difference (maybe over 100 000 volts) is applied between the cathode and the tungsten anode. The very fast electrons strike the anode, are absorbed and some of their energy is converted to X-rays, which are

sent in a direction determined by the angle of the tungsten target. (See figure 23.17.)

(b) The thin filter removes the soft X-rays, which are not used for imaging but expose the patient to unnecessary X-radiation. The soft X-rays are lower-energy X-rays.

5. (a) In a coherent bundle the fibres are in the same position relative to one another at each end of the fibre.

(b) A coherent bundle of optic fibres is used to transmit the reflected light back from the inside of the body to the eye or camera outside the body. It is necessary that an accurate image is produced and hence each fibre must remain in the same position relative to the other fibres. In this way light from the far right of the object will emerge from the body and be at the far right of the image; and this matching of light beams will occur for each optic fibre. The result will be an image that is exactly the same as the object inside the body.

(c) It is dark inside the body, hence the inside of the body will not be visible unless light is passed from a powerful light source along an optic fibre, reflected off the inside of the body and transmitted back to the outside. The internal organs are not necessarily good reflectors of light, so the source must be bright to reflect as much light as possible and see the organs.

(d) If the fibres in the bundle are narrow and have a large core to cladding ratio, light reflected from many points on the object will be transmitted. Hence the resulting image will be made of many points and will therefore be clearer.

6. Lasers are beams of electromagnetic radiation of very intense energy. Hence they can be used to cut through parts of organs and blood vessels. The beam can be made very narrow, allowing very precise cutting of tissue or destruction of tissue without damage to surrounding material.

7. $Z = \rho v$ where $Z = 1.59 \times 10^6$ kg m^{-2} s^{-1}, $v = 1570$ ms^{-1}

$\rho = 1.59 \times 10^6 \div 1570 = 1.01 \times 10^3$ kg m^{-3}

The density of blood is 1.01×10^3 kg m^{-3} (or 1.01 g cm^{-3}).

8. (a) $Z = \rho v$ where $\rho = 1060$ kg m^{-3}, $v = 1540$ m s^{-1}

$Z = 1060 \times 1540$

$= 1.63 \times 10^6$

The acoustic impedance of soft tissue is 1.63×10^6 kg m^{-2} s^{-1}.

(b) $Z = \rho v$ where $\rho = 1600$ kg m^{-3}, $v = 4080$ m s^{-1}

$Z = 1600 \times 4080 = 6.53 \times 10^6$

The acoustic impedance of bone is 6.53×10^6 kg m^{-2} s^{-1}.

(c) $I_r \div I_0 = (Z_2 - Z_1)^2 \div (Z_2 + Z_1)^2$ where $Z_1 = 1.65 \times 10^6$, $Z_2 = 1.71 \times 10^6$

$I_r \div I_0 = 0.000319$

9. (a) There is the most reflection at the skin–air interface (as I_r/I_0 has the largest value).

(b) There is the least reflection at the brain–fat interface (as I_r/I_0 has the smallest value).

(c) The greatest amount of absorption occurs where there is least reflection. This is at the brain–fat interface.

(d) For the fat–bone interface, $I_r/I_0 = 0.029$ where $I_0 = 60$ mW cm^{-2}

$\therefore I_r = 1.74$

The intensity of the reflected signal is 1.74 mW cm^{-2}.

(e) For the fat-muscle interface, $I_r/I_0 = 0.011$ where $I_0 = 80$ mW cm^{-2}

$\therefore I_r = 0.88$ mW cm^{-2}

Amount of signal travelling into the muscle = 80 – 0.88 mW cm^{-2}

= 79.12 mW cm^{-2}

(f) Some of the energy is converted to heat as the ultrasound travels through the muscle. The rest of the energy travels through the muscle to the next interface, where part is reflected and part is transmitted.

10. If the ultrasound travels through fat to liver, the same percentage of ultrasound will be reflected as when the ultrasound travels through liver to fat. Because of the squared term in the equation to find I_r/I_0 , it does not matter whether fat or liver is assigned the value Z_1, the ratio will remain the same.

11. A full bladder pushes the lower intestine, which contains a lot of gas, out of the way and lifts the uterus to a good position for taking an image. If the lower intestine were in the way, ultrasound would be reflected from the gas, and it would not be possible to obtain an image of the foetus.

12. $I_r \div I_0 = (Z_2 - Z_1)^2 \div (Z_2 + Z_1)^2$ where $Z_1 = 1.5 \times 10^6$ (for aqueous humour)

$Z_2 = 1.85 \times 10^6$ (for the lens) $= 0.011$

$I_r = 0.011 \times 15$

$= 0.16$

Intensity of the reflected beam is 0.16 mW cm^{-2}.

13. A continuous signal is directed at the foetal heart, and the reflected signal is detected by a separate receiver. The output is electronically filtered so that only the difference in frequency due to the Doppler shift is amplified. This difference is in the audible range. The tone will vary with the speed of

movement of the heart and hence with the heartbeats which can therefore be monitored.

14. Ultrasound signal is sent through heel bone; the transmitted ultrasound signal is received; the time of transmission is measured; the attenuation of the signal is recorded. This process is repeated many times, then the signal is analysed and compared with standards for normal bone.

15. (a) Attenuation, or reduction in intensity, of X-rays depends on the atomic density of the material encountered. Atomic density refers to the number of protons in the nuclei of the atoms encountered by the X-rays. Bone has a high atomic density so will absorb the X-rays readily and show up clearly in an X-ray image. Soft tissue has a moderate atomic density so will not absorb X-rays as well as bone. Soft tissue such as muscle and skin will show up very faintly on an X-ray image. Air, on the other hand has a low atomic density, so air in the body will not absorb X-rays and will appear black on an X-ray image.

    (b) The bone would be white, the muscle would be light grey and translucent in appearance, and the air would be black. Bone would absorb X-rays, because bone has a high atomic density as outlined in part (a), so the X-rays would not reach the photographic film, which would show up as white behind the bone. Some X-rays would penetrate the muscle, with a medium atomic density, and reach the film, resulting in the light grey colour behind muscle. X-rays would not be stopped by air, and these X-rays would reach the photographic film, resulting in a black image.

16. (a) Hard X-rays have more energy and greater penetrating power than soft X-rays.

(b) Because hard X-rays have greater penetrating power, they can pass through the muscles and skin and be absorbed by bone. Hence they allow an image of bones to be produced.

17.

Situation using CAT scan	**Problem with X-rays**	**Problem with ultrasound**
Soft tissue accurately imaged, resulting in fine detail being shown.	Soft tissue does not absorb conventional X-rays to any noticeable extent, so soft tissue will not show up, unless a contrast medium is used.	Ultrasound will not clearly distinguish one type of soft tissue from another.
Images of slices, close together, through the body can be taken.	Conventional X-rays cannot be used to image a slice of the body.	Ultrasound cannot accurately images slices close together.
The images of slices can be built up into a 3-dimensional image.	X-rays cannot do this as slices cannot be imaged.	Ultrasound cannot accurately image slices close together, so a 3-dimensional image built up from ultrasound is not very clear.
The brain can be imaged through the bony skull.	Conventional X-rays would show up the skull rather than what was	Ultrasound would be reflected from the skull and not image what was

	inside the skull.	underneath, unless the ultrasound was taken through a gap in the skull.
Can be used to investigate soft tissue damage where a contrast medium cannot be used.	X-rays will not show up soft tissue in this situation.	Ultrasound will not show enough contrast between soft tissues to detect tissue damage.
Can be used to scan the kidneys to obtain resolution better than 1 mm.	X-rays will not give clear images of the kidneys.	Ultrasound will not give this resolution-the resolution is determined by the wavelength of the ultrasound. In addition, the difference in intensity of the reflected ultrasound signals will not be great enough to give a clear image.
CAT scans provide a clear image of the lungs	X-rays are quicker but do not provide as clear an image.	Ultrasound cannot image the lungs because the ultrasound is strongly reflected at any interface where there is air.
Complicated bone structure is imaged and 3-dimensional image may be	Bone absorbs most of the rays giving a 2-dimensional image of the	Ultrasound is reflected from bone, so no useful image is formed.

obtained.	part of the bone facing the X-rays, so the complex structure cannot be seen	

18. The endoscope can be inserted into the oesophagus through the mouth. The tumour in the oesophagus can be viewed because light is reflected off the oesophagus and the tumour. A small tool on the end of the endoscope can be used to remove a sample of the tumour. This sample is then removed from the oesophagus by this tool and sent for examination. ‘Taking a biopsy’ means removing a sample of the tumour.

19. Keyhole surgery is less invasive than open surgery. The patient is more likely to recover quickly as the incision, if needed, is small, and there is less risk of infection, less swelling and less scarring. Often keyhole surgery can be carried out in an out-patients department, reducing the cost to the health system.

20. (a) Most of the ultrasound would be reflected from the interface between air and skin as the difference in acoustic impedance between air and skin is very large.

    (b) The optimum acoustic impedance of the gel would be the same as that of the skin. Then all the incident ultrasound would be transmitted into the skin.

    $Z_{gel} = 1010 \times 1540 = 1.56 \times 10^6$ kg m^{-2} s^{-1}

    (c) $Z = \rho v$ where $Z = 1.56 \times 10^6$ kg m^{-2} s^{-1}, $\rho = 1200$ kg m^{-3}

    $\therefore v = 1300$

    The speed of ultrasound in the gel is 1300 m s^{-1}.

21. (a) Speed of sound = 1540 m s^{-1}

Distance travelled by pulse = 700 mm = 7×10^{-1} m

Time for pulse to travel this distance = $\frac{0.7}{1540}$ s = 4.5×10^{-4} s

$\therefore$ minimum time between pulses = 4.5×10^{-4} s

(b) A faster pulse rate would mean that the time between pulses was shorter than 4.5×10^{-4} s. A new pulse would be sent into the body before the reflected pulse was received, leading to interference between the reflected pulse and the incoming pulse and making the image unclear.